Learning Dynamic Causal Models

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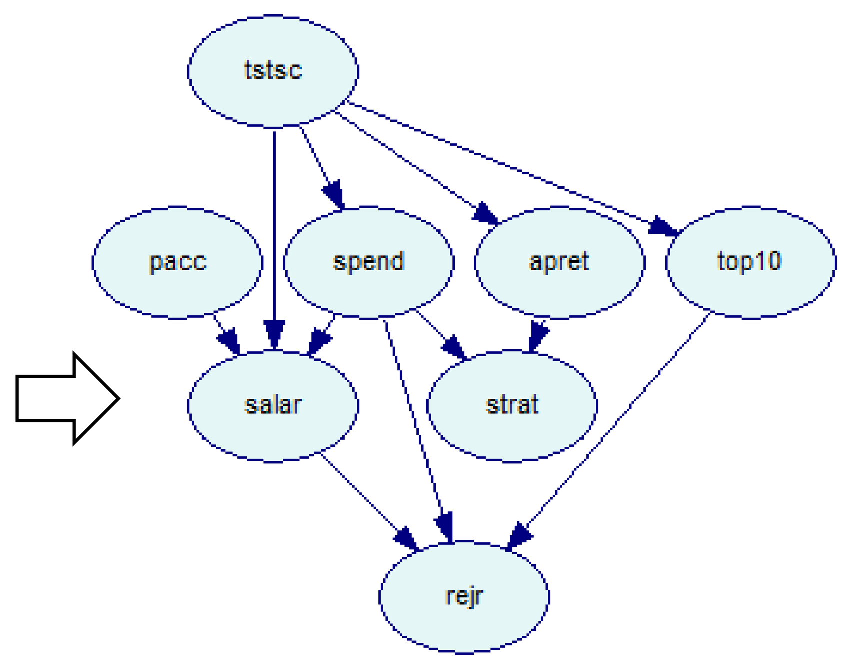
1. Causal Discovery

Find a causal graph that could have generated the data.

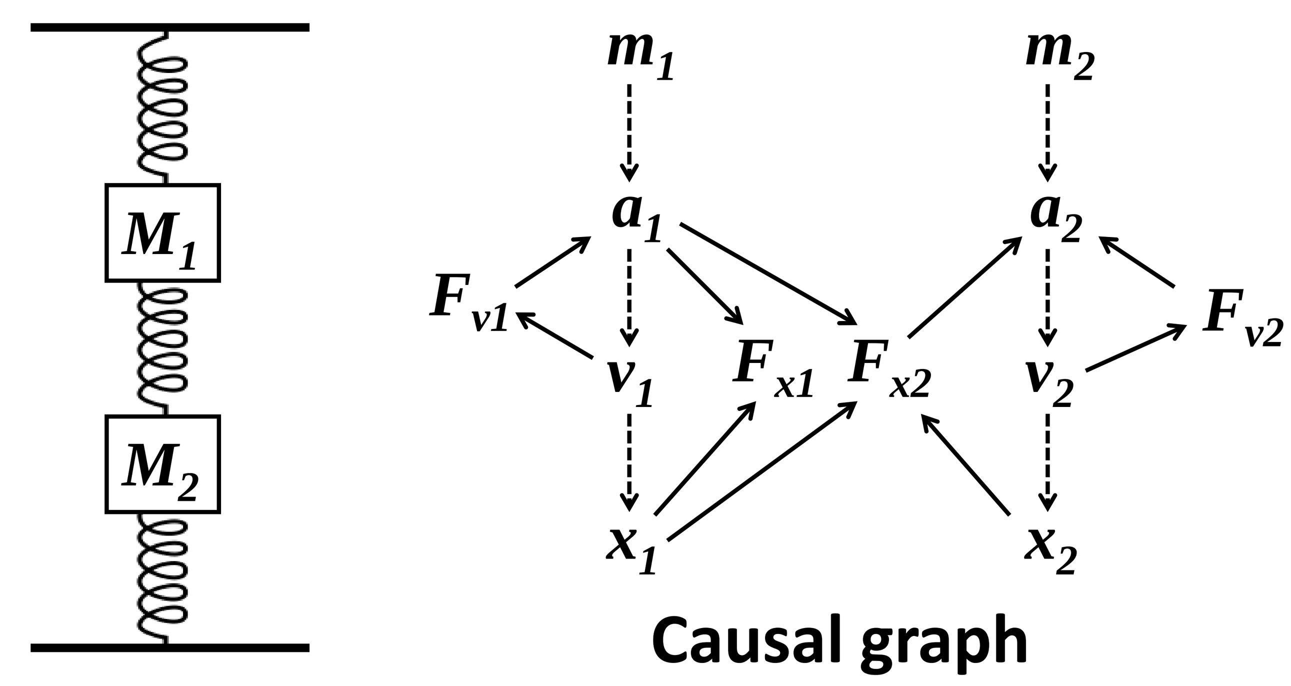
spend	apret	top10	rejr	tstsc	pacc	strat	salar
9855	52.5	15	29.474	65.063	36.887	12	60800
10527	64.25	36	22.309	71.063	30.97	12.8	63900
7904	37.75	26	25.853	60.75	41.985	20.3	57800
6601	57	23	11.296	67.188	40.289	17	51200
7251	62	17	22.635	56.25	46.78	18.1	48000
6967	66.75	40	9.718	65.625	53.103	18	57700
8489	70.333	20	15.444	59.875	50.46	13.5	44000
9554	85.25	79	44.225	74.688	40.137	17.1	70100
15287	65.25	42	26.913	70.75	28.276	14.4	71738
7057	55.25	17	24.379	59.063	44.251	21.2	58200
16848	77.75	48	26.69	75.938	27.187	9.2	63000
18211	91	87	76.681	80.625	51.164	12.8	74400
21561	69.25	58	44.702	76.25	26.689	9.2	75400
20667	65	68	22.995	75.625	28.038	11	66200
10684	61.75	26	8.774	66	33.99	9.5	52900
11738	74.25	32	25.449	66.875	27.701	12	63400
10107	74	43	11.315	71	29.096	16.2	66200
7817	65.75	36	33.709	64.25	52.548	17.7	54600
7050	26	11	0	55.313	55.651	18.8	59500
9082	83.5	73	64.668	77.375	43.185	13.6	66700
11706	60	56	16.937	73.75	39.479	12.7	62100
7643	49.25	23	36.635	62.813	39.302	18.7	57700
25734	90	77	67.758	80.938	44.133	10	80200



apply to

2. Dynamic Systems

(e.g., a coupled harmonic oscillator)



Causal graph

3. Motivation

1. The causal structure of a dynamic system can change as the time-scale of observation of the system is increased [Iwasaki and Simon 1994]. (Variables may equilibrate over time.)
2. One of the consequences of this fact is that causal reasoning in equilibrium models may be incorrect [Dash 2003] (A manipulation takes the system out of equilibrium.)
   I.e., $Equil(Do(M, \mathbf{U} = \mathbf{u}), X) \neq Do(Equil(M, X), \mathbf{U} = \mathbf{u})$

=> Therefore, we should learn causal dynamic models.

4. Representation

Difference-based causal models:

1. *All causation across time results from a derivative causing a change in its variable.*
2. *Highest order derivatives are caused contemporaneously.*

Based on the Iwasaki-Simon [1994] representation and motivated by real physical systems, such as the coupled harmonic oscillator in the example.

5. Algorithm (2 steps)

Use **time series** to:

1. Find the highest order derivatives (called prime variables):

   Theorem 1 (detecting prime variables). *Let I be the set of conditional independence relations implied by faithfulness applied to a DBCM $M = \langle \boldsymbol{V}, \boldsymbol{E} \rangle$, where $\boldsymbol{V} = \{\boldsymbol{V'}, \boldsymbol{V_\Delta}\}$. Then let $\Delta^j V_i$ denote the difference of some $V_i \in \boldsymbol{V'}$. Then $\Delta^j V_i$ is the prime variable of V_i if and only if:*

   1. *there exists a $W \subset \boldsymbol{V}$ such that $(\Delta^j V_i^t \perp\!\!\!\perp \Delta^j V_i^{t+1} \mid W) \in I$, and*
   2. *there is no set $W' \subset \boldsymbol{V}$ such that $(\Delta^k V_i^t \perp\!\!\!\perp \Delta^k V_i^{t+1} \mid W') \in I$ for all $k < j$.*

2. Find the contemporaneous structure:

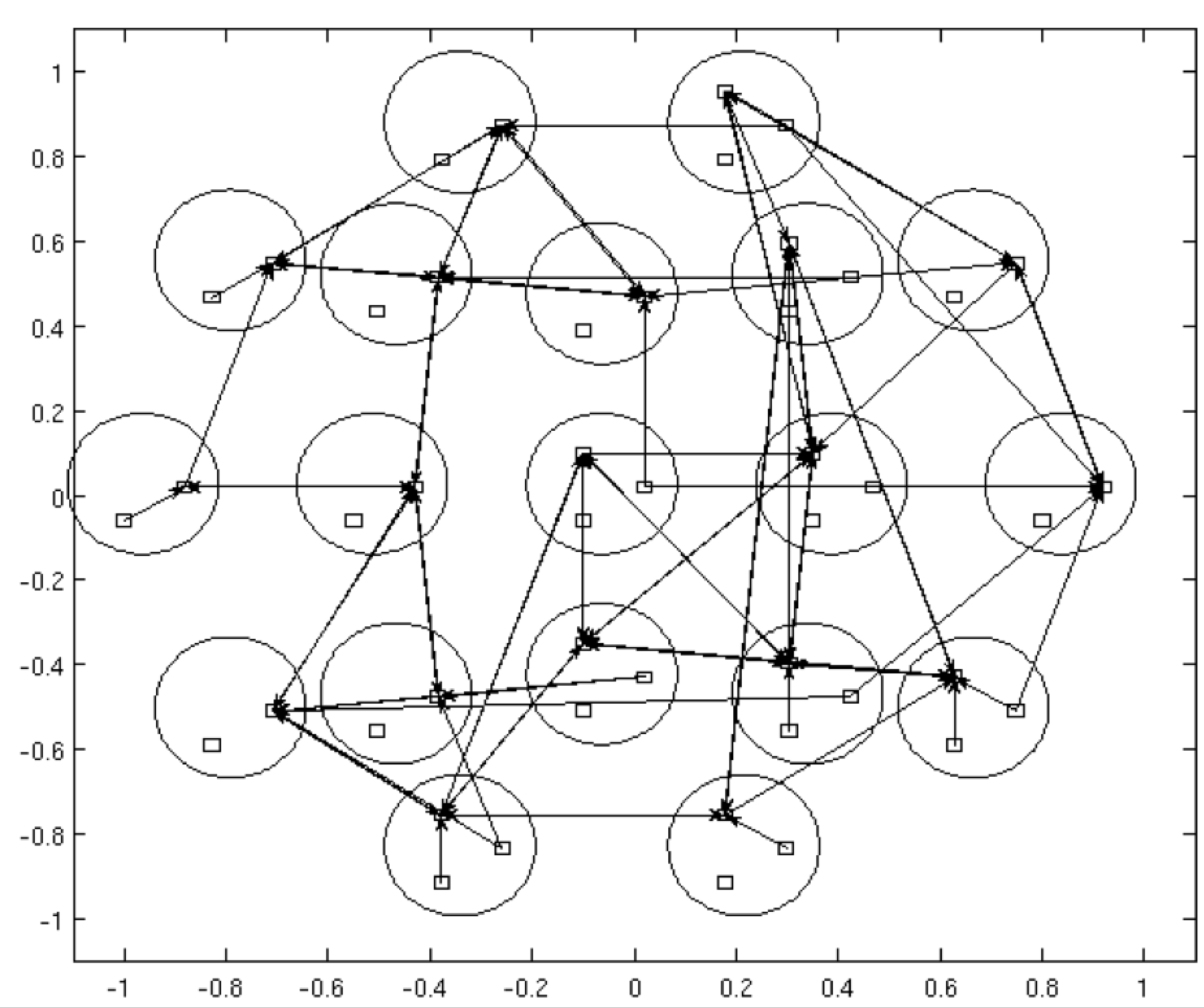
   Theorem 2 (learning contemporaneous structure). *Let I be the set of conditional independence relations implied by faithfulness applied to a DBCM $M = \langle \boldsymbol{V}, \boldsymbol{E} \rangle$, where $\boldsymbol{V} = \{\boldsymbol{V^0}, \boldsymbol{V^1}\}$ There is an edge $V_1^t - V_2^t$ if and only if there exists no $\boldsymbol{V'}^1 \subset \boldsymbol{V}^1 \setminus \{V_1^1, V_2^1\}$ such that $(V_1^1 \perp\!\!\!\perp V_2^1 \mid \boldsymbol{V'}^1) \in I$.*

6. Experimental Results

Generated data from the coupled harmonic oscillator and tried to relearn it:

1. All the correct highest derivatives were found.
2. All the contemporaneous edges were found correctly, but two orientations were incorrect. This results from a violation of faithfulness, but at the moment I am not sure exactly why this violation occurs.

I also applied the algorithm to EEG data in a preliminary study to find a causal structure of the alpha waves in the human brain. The result is shown in the figure below, where every circle is a brain region. We would expect connections between neighboring brain regions, which is the case.



Acknowledgements

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