The causal structure of a dynamic system can change as the time-scale of observation of the system is increased [Iwasaki and Simon 1994]. (Variables may equilibrate over time.) One of the consequences of this fact is that causal reasoning in equilibrium models may be incorrect [Dash 2003] (A manipulation takes the system out of equilibrium.) I.e., \( \text{Equil} Do(M, U = u), X \neq Do(\text{Equil}(M, X), U = u) \) => Therefore, we should learn causal dynamic models.

**1. Causal Discovery**
Find a causal graph that could have generated the data.

**2. Dynamic Systems**
(e.g., a coupled harmonic oscillator)

**3. Motivation**
1. The causal structure of a dynamic system can change as the time-scale of observation of the system is increased [Iwasaki and Simon 1994]. (Variables may equilibrate over time.)
2. One of the consequences of this fact is that causal reasoning in equilibrium models may be incorrect [Dash 2003] (A manipulation takes the system out of equilibrium.)

I.e., \( \text{Equil} Do(M, U = u), X \neq Do(\text{Equil}(M, X), U = u) \)

=> Therefore, we should learn causal dynamic models.

**4. Representation**

Difference-based causal models:
1. All causation across time results from a derivative causing a change in its variable.
2. Highest order derivatives are caused contemporaneously.

Based on the Iwasaki-Simon [1994] representation and motivated by real physical systems, such as the coupled harmonic oscillator in the example.

**5. Algorithm (2 steps)**

Use time series to:
1. Find the highest order derivatives (called prime variables):

   **Theorem 1** (detecting prime variables). Let \( I \) be the set of conditional independence relations implied by faithfulness applied to a DBCM \( M = (V, E) \), where \( V = \{ V^1, V^2 \} \). Then let \( \Delta V^i \) denote the difference of some \( V^i \in V^i \). Then \( \Delta V^i \) is the prime variable of \( V^i \) if and only if:

   1. there exists a \( W \subset V \) such that \( \Delta V^i \perp \Delta V^{i+1} \mid W \) \( \in I \), and
   2. there is no set \( W^* \subset V \) such that \( \Delta V^j \perp \Delta V^{j+1} \mid W^* \) \( \in I \) for all \( k < j \).

2. Find the contemporaneous structure:

   **Theorem 2** (learning contemporaneous structure). Let \( I \) be the set of conditional independence relations implied by faithfulness applied to a DBCM \( M = (V, E) \), where \( V = \{ V^1, V^2 \} \). There is an edge \( V^i \rightarrow V^j \) if and only if there exists no \( V^l \subset V^i \setminus \{ V^1, V^2 \} \) such that \( (V^1 \perp V^2 \mid V^l) \in I \).

**6. Experimental Results**

Generated data from the coupled harmonic oscillator and tried to relearn it:
1. All the correct highest derivatives were found.
2. All the contemporaneous edges were found correctly, but two orientations were incorrect. This results from a violation of faithfulness, but at the moment I am not sure exactly why this violation occurs.

I also applied the algorithm to EEG data in a preliminary study to find a causal structure of the alpha waves in the human brain. The result is shown in the figure below, where every circle is a brain region. We would expect connections between neighboring brain regions, which is the case.